

Why a Negative Times a Negative Equals a Positive

By Adrian Apollo

Consider:

$7 - 4$ ("Take four away from seven"). It leaves you the same result as $7 + -4$ ("Add a debt of four to seven" or "Add a taking away of four to seven"). Now consider the three debts of four dollars, as indicated in this expression: $7 + -4 + -4 + -4$, which means the same as "add three four-dollar debts to your original balance of seven dollars." Another way to write that is $7 + 3(-4)$.

Now let's suppose you had 40 dollars in your pocket and you also had three five-dollar debts. Your net worth would be: $40 + 3(-5) = 25$ dollars. Now starting with your net worth of 25 dollars, let's remove one of the five-dollar debts. Since "removing" means "subtract a debt," we can calculate our new net worth as being $25 - (-5) = 30$ dollars.

Let's turn a verb ("remove") into its noun form ("removal") by adding "a removal" of a debt of five dollars to 25. In math language, that is: $25 + -(-5) = 30$ dollars. Another way to look at it is "adding *one* removal," i.e., $25 + (-1)(-5) = 30$ dollars.

So we see that $(-3)(-5)$ ("three removals of a five-dollar debt" or "three removals of a subtracting of five dollars") results in the *addition* of 15 dollars to our net worth, and so therefore, a negative integer times a negative integer produces a positive integer.

Positive integers can be seen as being noun forms of the verb "to add." So, for example, $25 + +15$ ("25 plus positive 15" or "add an adding of 15 to 25") results in the same thing as $25 + 15$ ("add 15 to 25").

So, positive 15 (the integer) is not the same thing as plain 15 (the natural number), because "positive 15" means "the adding of 15" and "plain 15" just means 15. The two ideas are different, just as the phrase "drinks water" is not the same thing as the word "water," even though both ideas involve the concept of water. "Plain 15" refers simply to 15 things, just like a collective noun. "Positive 15" refers to an action being taken, and that action is expressed in a noun form, just as the noun form of "drink" could be expressed as "the drinking of."

What confuses people who try to make sense out of why a negative integer times a negative integer equals a positive integer is that they think that a plain number is interchangeable with its corresponding integer. It is not. Then they scratch their heads, wondering why the multiplication of two "negative quantities" (things that seemingly don't exist) gives a plain number, which refers to something that *does* exist.

It's as if two non-things multiply to give a thing. This is the same thing as confusing "drinks water" with "water." The two are highly similar and related, but they are not identical or interchangeable.

Of course, -2, for example, does not represent two "non-things." It means "the taking away of 2 things" or "the leftward movement of two units" on a number line. In the second case, we see how the concepts of "positive" and "negative" refer to the idea of direction.

Integers, then, can be seen as being one-dimensional vectors-an insight that is rarely referred to by mathematicians. Even though the one-dimensional vector has only one component (in the way that the term "component" is normally used in math), that one component itself is made up of two components of a different kind, the negative component (the directionality) and the quantity component (the 2).

The only exception to this analysis would be the integer zero, but that's a whole 'nother story. "Zero" is really just a figure of speech. It means: "the absence of quantity." It's a concept of method, not a concept of entities.

(Posted: March 26, 2003)